Why is portfolio insurance attractive to investors?

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Abstract

This paper examines whether and how the popularity of portfolio insurance strategies can be justified theoretically. It analyzes the two most popular portfolio insurance strategies, option based portfolio insurance and constant proportion portfolio insurance. The analysis is done both for an investor with constant relative risk aversion and for a cumulative prospect theory investor. We employ three different return generating processes with and without stochastic volatility and jumps. We find that a CRRA investor does not profit from portfolio insurance and chooses rather low protection levels if forced to use it. A CPT investor, on the other hand, strongly prefers portfolio insurance to constant proportion strategies, with the certainty equivalent return from trading doubling from around 5% for constant proportion to around 10% due to portfolio insurance. Both loss aversion and probability weighting turn out to be crucial to explain the attractiveness of portfolio insurance, and utility gains drop sharply if one of these two elements of CPT is eliminated. While the overall attractiveness of portfolio insurance holds in all models, the choice between constant proportion portfolio insurance and option based portfolio insurance depends on the return generating process.

Keywords: cumulative prospect theory, portfolio planning, portfolio insurance, stochastic volatility, stochastic jumps

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1 Introduction and Motivation

Portfolio insurance (PI) strategies belong to the most prominent investment strategies within the US market for structured products (see e.g. Hens and Rieger (2009)). They account for a significant share of this growing segment (see Pain and Rand (2008)), with decreasing importance in upward moving markets and increasing importance after market downturns. Subsequent to the huge stock market losses in 2008 sales of capital protected products have significantly increased in the first half of 2009 and command a market share of around 40%.[1] By far the two most popular PI strategies are option based portfolio insurance (OBPI) and constant proportion portfolio insurance (CPPI). OBPI originated shortly after the seminal article of Black and Scholes (1973), when Leland and Rubinstein (1976) pioneered to suggest put options for hedging portfolios. CPPI was introduced by Perold (1986) for bond instruments and Black and Jones (1987) for equity instruments.

In this paper, we analyze PI strategies from the point of view of an individual investor. We assess the relative attractiveness of CPPI and protective put strategies, as one example of OBPI, and compare them to conventional investment strategies like buy-and-hold or constant proportion. The analysis is done both for an investor with constant relative risk aversion (CRRA) and for a cumulative prospect theory (CPT) investor. We do not limit the analysis to a Black-Scholes setup, but allow for stochastic volatility and stochastic jumps in the stock price, like in the models of Heston (1993) or Bakshi, Cao, and Chen (1997). This allows us to assess the performance of the strategies in a setup where investment opportunities are stochastic.

We find that CPT preferences are perfectly able to explain the attractiveness of PI. While a CRRA investor does not choose PI strategies, the combination of loss aversion and non-linear probability weighting in CPT preferences leads to a strong preference for PI strategies, with the gain in the certainty equivalent from trading doubling from around 5% for constant proportion strategies to around 10% for PI strategies. The attractiveness of PI holds for all return generating processes, while the model setup determines whether CPPI or OBPI are the overall optimal choice.

Our basic question is whether and how the popularity of portfolio insurance strategies with investment companies and retail investors can be justified theoretically. Intuitively, a PI strategy should be attractive for an investor with a strong incentive to avoid wealth levels below a given guarantee. For a risk averse investor who maximizes expected utility, marginal utility increases if the wealth level de-

[1] US market data is provided by www.StructuredRetailProducts.com.

creases. However, this gradual increase is not large enough to favor PI strategies.[2] We thus consider cumulative prospect theory (CPT), currently the most influential descriptive model of decision under risk.[3] It was developed by Tversky and Kahneman (1992) in their seminal paper and accounts for three well documented deviations from the normative expected utility benchmark: probability weighting, loss aversion and non-monotonic curvature (concavity/convexity). Intuitively, the first two features are the reason that avoiding losses is more important for a CPT investor than for an investor maximizing expected utility. The question then is whether CPT preferences indeed induce the investor to favor PI strategies.

Note that we do not aim at finding the overall optimal strategy for given preferences of the investor.[4] We rather restrict our analysis to simple and straightforward strategies which are easy to describe and to implement. The investor can follow CPPI strategies, protective put strategies as the most popular example of OBPI, as well as simple buy-and-hold and constant proportion strategies. We then determine the optimal choice of a CRRA investor and a CPT investor within each strategy class and over all of the investment strategies considered. We aim at assessing the conditions for PI strategies to be optimal, and to identify the features of the utility function which explain the attractiveness of PI.

Our main findings are as follows: For a CRRA investor, the optimal choice is in most cases given by constant proportion strategies. The utility differences between following the optimal constant proportion strategy, the optimal buy-and-hold strategy or the optimal PI strategy (for which the optimal protection level is well below the initial wealth) are, however, rather small. While a PI strategy with a low protection level is still a sensible choice for a CRRA investor, it is neither the overall optimal strategy nor superior to the simple strategies. The attractiveness of PI strategies is thus puzzling as long as we stick to the assumption of expected utility and CRRA.

For a CPT investor, the picture changes. The utility gains of PI are large and

[2]A PI strategy is optimal in the case of a subsistence level and a utility of minus infinity for a wealth below this subsistence level. Here, we rather analyze a situation where the investor has already invested the present value of the subsistence level at the risk-free rate and now deals with the optimal strategy for investing his additional wealth (for which he will still survive a total loss).

[3]See e.g. Abdellaoui, Bleichrodt, and L'Haridon (2008).

[4]For a CRRA investor, the optimal terminal payoff in a model with stochastic volatility and jumps, e.g., depends on the whole path of (latent) volatility. To achieve this payoff with standard contracts, the investor needs to trade continuously in infinitely many options or to accept a payoff which depends on latent variables. For a CPT investor, overall optimal strategies turn out to be rather extreme, in that the investor aggressively gambles for very high returns.

economically significant, and either CPPI strategies or protective puts are optimal. The optimal protection level is in most cases slightly above the initial wealth and thus slightly above the reference point. With CPT preferences, the investor indeed has a strong incentive to avoid even small losses. Among the PI strategies we document a slight preference for CPPI, which has a larger upside potential than OBPI strategies. The attractiveness of PI strategies can thus be explained by CPT. In a next step, we perform a sensitivity analysis for a CPT investor to assess the importance of curvature and loss aversion in the utility function and of non-linear probability weighting for explaining the attractiveness of PI. With probability weighting and loss aversion, PI is highly superior to CP with utility gains of around 550 basis points. If only one component is present, however, the investor's profits from PI drop sharply to about 80 basis points. The absence of curvature (resulting in a linear value function with a kink at the reference point) does not erode the comparative advantage of PI.

The analysis is done in the model of Black and Scholes (1973), the stochastic volatility model of Heston (1993), and the model of Bakshi, Cao, and Chen (1997) with both stochastic volatility and jumps in the stock price. Overall results are similar across the various models, so that neither stochastic investment opportunities due to stochastic volatility nor jumps change the relative attractiveness of PI strategies.[5] However, we find that the relative attractiveness of CPPI versus OBPI crucially depends on the return generating process and on the parameters of the preferences. In particular, jumps induce the investor to prefer CPPI to OBPI.

Our paper is related to several strands of the literature on portfolio planning. The first strand analyzes the optimal strategies in case of expected utility with no (or only technical) restrictions on admissible strategies for various utility functions and model setups. For a CRRA investor, the optimal strategy in a Black-Scholes economy is determined by Merton (1971). Liu, Longstaff, and Pan (2003) find the optimal strategy in a model with stochastic volatility and jumps if only the stock and the bond are traded, while Liu, Longstaff, and Pan (2003) and Branger, Schlag, and Schneider (2008) extend the results to a complete market. In a second strand of the literature, the optimal investment strategy for a CPT investor is considered. Berkelaar, Kouwenberg, and Post (2004) and Gomes (2005) analyze CPT without probability weighting and Jin and Zhou (2008) include probability weighting. A

[5]Note that we do not allow for more complex strategies. In particular, the investor cannot trade continuously in derivatives. As shown by Branger, Breuer, and Schlag (2009), trading in derivatives can lead to large differences between the BS model and models with more risk factors.

CPT investor's optimal payoff turns out to be rather extreme, in that the investor pursues an aggressive gambling policy betting on good states of the market. A third strand of the literature analyzes the preferences an investor must have for PI being the optimal choice. For expected utility, we refer to Merton (1971), Leland (1980), Brennan and Solanki (1981), Benninga and Blume (1985), Perold and Sharpe (1988), Black and Perold (1992) and Karatzas and Shreve (1998).[6] Brandt and Aït-Sahalia (2001), Berkelaar, Kouwenberg, and Post (2004), and Hens and Rieger (2009) show that assuming loss aversion helps to explain the attractiveness of PI. Dierkes, Erner, and Zeisberger (2009) additionally consider probability weighting and find it to be the driving factor for PI strategies' attractiveness under CPT. They rely on time-series data for the S&P 500 and T-bills, and show that the optimal strategy strongly depends on the length of the investment horizon. In contrast, our focus is a comparison of optimal strategies and the assessment of the size and the drivers of the utility gains from using PI. The fourth strand of the literature analyzes PI in non Black-Scholes setups. Betrand and Prigent (2003) compare the terminal value from OBPI and CPPI in a model with stochastic volatility. Cont and Tankov (2009) focus on gap risk due to jumps in the stock price and analyze the pricing of an insurance against this risk.

The remainder of the paper is organized as follows. In Section 2, we introduce the model and the setup of the simulation study and specify the portfolio planning problem. Section 3 gives the results of the analysis and compares the attractiveness of various strategies under both CRRA and CPT. Section 4 concludes.

2 Model Setup

2.1 The Model

The analysis is carried out in a partial equilibrium model. We take the price processes of the assets as given and do not consider equilibrium implications of PI or try to

[6]PI strategies arise from a subsistence level or from the exogenous constraint that the value of the strategy is above a specified wealth level. For this line of research we refer to the works of Cox and Huang (1989), Brennan and Schwartz (1989), Grossman and Villa (1989), Grossman and Zhou (1993), Grossman and Zhou (1996), Basak (1995), Cvitanic and Karatzas (1995), Cvitanic and Karatzas (1999), Browne (1999), Tepla (2000), Tepla (2001) and El Karoui, Jeanblanc, and Lacoste (2005). For an overview, see Balder and Mahayni (2009).

answer the question which market participants will supply PI for which reasons.[7]

We consider the SVJ-model of Bakshi, Cao, and Chen (1997) and Bates (1996) with stochastic volatility and jumps in the stock price, but constant interest rates. The dynamics of the stock price (or index level) under the physical measure $\mathbb{P}$ are given by

$$\begin{aligned} dS_t &= (r + a_t - \bar{\mu}^{\mathbb{P}}\lambda^{\mathbb{P}})S_t dt + \sqrt{V_t} S_t dW_t^{S,\mathbb{P}} + \left(e^{X_t} - 1\right) S_{t-} dN_t \\ dV_t &= \kappa^{\mathbb{P}}(\theta^{\mathbb{P}} - V_t) S_t dt + \sqrt{V_t}\sigma_V \left(\rho dW_t^{S,\mathbb{P}} + \sqrt{1-\rho^2} dW_t^{V,\mathbb{P}}\right). \end{aligned}$$

$W_t^{S,\mathbb{P}}$ and $W_t^{V,\mathbb{P}}$ are independent Wiener processes. N_t is a Poisson process with constant intensity $\lambda^{\mathbb{P}}$. The jump size X_t in the (log) stock return follows a normal distribution: $X_t \sim \mathcal{N}\left(\mu_S^{\mathbb{P}}, (\sigma_S^{\mathbb{P}})^2\right)$, and the mean jump size in the stock price is

$$\bar{\mu}^{\mathbb{P}} = e^{\mu_S^{\mathbb{P}} + 0.5(\sigma_S^{\mathbb{P}})^2} - 1.$$

We assume that there are no dividend payments. The expected excess return a_t on the stock is given by

$$a_t = \eta_S V_t + \lambda^{\mathbb{P}}\bar{\mu}^{\mathbb{P}} - \lambda^{\mathbb{Q}}\bar{\mu}^{\mathbb{Q}}. \tag{1}$$

It can be decomposed into a premium $\eta_S V_t$ for stock diffusion risk and a premium for stock jump risk. The latter depends on the jump intensity $\lambda^{\mathbb{Q}}$ and the mean jump size $\bar{\mu}^{\mathbb{Q}}$ under the risk-neutral measure $\mathbb{Q}$ and on the same quantities under the true measure $\mathbb{P}$.

The dynamics under the risk-neutral measure $\mathbb{Q}$ are

$$\begin{aligned} dS_t &= (r - \bar{\mu}^{\mathbb{Q}}\lambda_S^{\mathbb{Q}})S_t dt + \sqrt{V_t} S_t dW_t^{S,\mathbb{Q}} + \left(e^{X_t} - 1\right) S_{t-} dN_t \\ dV_t &= \kappa^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_t) S_t dt + \sqrt{V_t}\sigma_V \left(\rho dW_t^{S,\mathbb{Q}} + \sqrt{1-\rho^2} dW_t^{V,\mathbb{Q}}\right). \end{aligned}$$

The mean reversion speed and the mean-reversion level of the variance are given by

$$\begin{aligned} \kappa^{\mathbb{Q}} &= \kappa^{\mathbb{P}} + \eta_V \\ \kappa^{\mathbb{Q}}\theta^{\mathbb{Q}} &= \kappa^{\mathbb{P}}\theta^{\mathbb{P}}, \end{aligned}$$

where η_V denotes the premium for (total) volatility diffusion risk. The intensity of the jump process under $\mathbb{Q}$ is $\lambda^{\mathbb{Q}}$. For the jump size, we assume that it still follows a normal distribution, but that all parameters of this distributions may change:

$$X_t \sim \mathcal{N}\left(\mu_S^{\mathbb{Q}}, (\sigma_S^{\mathbb{Q}})^2\right).$$

[7] Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001) and Barberis and Xiong (2009) study the pricing implications of PT. For an economy with CPT investors Barberis and Huang (2008) show that due to probability weighting, a positively skewed security can become overpriced relative to the prediction of EUT, and can earn a negative average excess return.

The SVJ-model nests the other two option pricing models considered in our analysis. Setting $\lambda^{\mathbb{P}} = \sigma_V = 0$ and $V_t = \theta$ gives the Black-Scholes model. For the Heston (1993) (SV) model, we set $\lambda^{\mathbb{P}} = 0$.

The parameters for the physical measure are taken from Eraker, Johannes, and Polson (2003). Based on these parameters, Broadie, Chernov, and Johannes (2007) have estimated the parameters under the risk-neutral measure. We rely on their estimates in the following, which are summarized in Table 1.

2.2 Utility Functions

We assume that our investor derives utility from terminal wealth only. The planning horizon is equal to one year. This choice is in line with capital gains taxation, where taxes are paid on the gains over the last year.

For an investor who maximizes expected utility, we assume constant relative risk aversion (CRRA). The utility function is given by $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$ for $\gamma \neq 1$ and $u(w) = \ln w$ for $\gamma = 1$, where w is the terminal wealth and γ is the relative risk aversion. In the numerical analysis, we consider $\gamma \in [0.1; 10]$.

For cumulative prospect theory (CPT), we rely on the original parametric specification of Tversky and Kahneman (1992) for the value function

$$v(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -k(-x)^{\beta} & x < 0 \end{cases},$$

where x is the gain respectively loss relative to the reference point which is set equal to the initial wealth level of 100. For the standard choice, Tversky and Kahneman's median values of $\alpha = 0.88$ and $\beta = 0.88$, the value function is concave for gains and convex for losses. The parameter $k = 2.25$ induces a kink in the value function at $x = 0$, which results in pronounced loss aversion. Note that a capital gains tax with an asymmetric treatment of gains and losses also leads to a kink at the reference point, which is, however, much smaller than the one assumed here. In the following, we restrict the analysis to the case without taxes and implicitly subsume their impact in the loss aversion parameter.

The probability weighting function is given by

$$w^+(p) = \frac{p^{\delta+}}{(p^{\delta+} + (1-p)^{\delta+})^{\frac{1}{\delta+}}}$$

$$w^-(p) = \frac{p^{\delta-}}{(p^{\delta-} + (1-p)^{\delta-})^{\frac{1}{\delta-}}}.$$

In accordance with Tversky and Kahneman (1992) we set $\delta^+ = 0.61$ and $\delta^- = 0.69$. The inverse S-shape of the probability weighting function leads to an overweighting of extreme gains and losses, while moderate changes have a lower weight. Note that the 'probabilities' after transformation do not necessarily add up to one.

Although numerous other forms have been suggested, this original formulation is most often used. No alternative has proved to be clearly superior in subsequent experimental studies (see e.g. Abdellaoui, Bleichrodt, and L'Haridon (2008)). As a robustness check we also consider the probability weighting function of Rieger and Wang (2006) which is less steep in the extremes:[8]

$$w(p) = \frac{3-3b}{a^2-a+1}\left(p^3-(a+1)p^2+a\,p\right)+p$$

To ensure comparability to the parameterization of the Tversky and Kahneman (1992) probability weighting function, we estimate the parameters a and b via non-linear least squares from the median data of Tversky and Kahneman (1992) (Table 3, p. 307). This gives $a = -0.3797$ and $b = -0.1544$ for gains and $a = -0.4054$ and $b = -0.4773$ for losses.

To assess the utility improvement from the different strategies, we rely on the certainty equivalent. It is defined as the deterministic level of terminal wealth at time T which gives the investor the same utility as the uncertain wealth resulting from the strategy under consideration.

In case of expected utility, the certainty equivalent (CE) for a strategy i with terminal wealth $W_T^{(i)}$ is given by

$$\frac{(CE^{(i)})^{1-\gamma}}{1-\gamma} = E\left[\frac{(W_T^{(i)})^{1-\gamma}}{1-\gamma}\right],$$

where $W_T^{(i)}$ denotes the terminal wealth for strategy i. For CPT, the certainty equivalent follows from

$$CE^{(i)} = \left(CPT^{(i)}\right)^{\frac{1}{\alpha}},$$

where $CPT^{(i)}$ denotes the utility of the CPT investor.

2.3 Investment Strategies

We consider simple investment strategies which are easy to describe and to implement and which do neither involve continuous trading in derivatives nor state-dependent portfolio weights. Note that we do not aim at finding the overall optimal

[8] Alternatively, we could have adjusted δ^+ and δ^-, which would however not change the overall form of the weighting function.

strategies, but rather compare a set of given strategies including PI strategies. We want to assess whether PI strategies are superior to the standard strategies.

For all strategies, the minimum portfolio weight of the stock is 0%. Shorting the risky asset is not permitted as it would have little in common with the idea of PI. The maximum portfolio weight of the stock is 133%. This value corresponds to the allowed leverage for investment funds under the SEC 1940 Act [Section 18 (f) (1)].[9]

2.3.1 Buy-and-Hold, Constant Proportion

The easiest strategy we consider is a buy-and-hold (BH) strategy. At $t = 0$, the investor decides on how much to invest into the stock, and he does not adjust his portfolio afterwards. We assume that the initial portfolio weight of the stock varies between 0% and 133%. While the number of stocks remains constant over time, the portfolio weight of the stock will change over time, reflecting different rates of return on the stock and the risk-free asset.

In contrast, the weight of the stock stays constant over time (while the number of stocks is continuously adjusted) if the investor follows a constant proportion (CP) strategy. Here, the investor decides on the fraction of wealth to invest into the stock, and rebalances his portfolio continuously to keep the actual weight of the stock equal to the fixed weight. Again, we consider weights of the stock that vary between 0% and 133%.

2.3.2 CPPI

CPPI strategies are characterized by a floor, which is equal to the discounted value of a guaranteed terminal payoff, and a multiplier. If the current value of the portfolio is larger than the floor, the strategy invests a multiple of the difference (often referred to as 'cushion') into the stock, while the rest is invested or borrowed (the investor takes a loan) at the risk-free rate. Like for the CP strategy, the portfolio has to be rebalanced continuously. Under ideal conditions, i.e. continuous trading and a continuous stock price process, the terminal value of the strategy does never fall below the floor. If there are jumps or if the investor trades at discrete points in time

[9]Higher leverage is accessible at the level of the individual investor. The Board of Governors of the Federal Reserve System limits the extent to which stock purchases can be leveraged by an initial margin requirement of 50 percent. Again, very different regulatory requirements apply for hedge funds.

only, however, and the investor uses a multiplier above one, he can end up with less wealth than the floor, which is the so-called gap risk.

In our implementation, we consider protection levels between 50% and 100%. The multiplier is at least 0.1 and at most 10. Since we cap the portfolio weight at 133%, the maximum value of 10 is way less extreme than it seems at a first glance.

The properties of CPPI have been extensively studied in literature. References include Bookstaber and Clarke (1984), Bookstaber and Langsam (1988), Zhou and Kavee (1988), Black and Rouhani (1989), Benninga (1990), Black and Perold (1992), Betrand and Prigent (2005), Balder, Brandl, and Mahayni (2009), Balder and Mahayni (2009), and Zagst and Kraus (2009).

2.3.3 OBPI

Option-based portfolio insurance strategies are not based on continuous trading in the stock and a risk-free asset, but rely on derivatives. In a protective put strategy, the investor allocates his money to stocks and the same number of puts. The number of stocks is chosen such that the initial value of stocks and puts is just equal to the initial wealth. If the stock price falls below the strike price, then the value of the portfolio is equal to the number of stocks (or puts), multiplied by the strike price of the put, which defines the protection level.

In a variant of the protective put strategy (PP133), we assume that the investor allocates 133% of his wealth to the stock, buys the respective number of puts to protect this position, and finances the portfolio by taking a loan at the risk-free rate. In this variant the protective put strategy is on par with the other strategies regarding the maximum amount of leverage.

2.4 Simulation Setup

To assess the utility from our strategies, we rely on a Monte-Carlo simulation with 10,000 runs.[10] For each run, we simulate the whole path of the stock price, using an Euler discretization with 10 time steps per day. For each trading strategy, we then calculate the realized terminal wealth on each path and determine the certainty equivalents for both a CRRA and a CPT investor.

[10]The graphs are based on 1,000,000 runs.

3 Results

3.1 CRRA investor

The optimal strategies and the CEs for the CRRA investor with $\gamma = 3$ are given in Table 2. In the Black-Scholes model, the portfolio planning problem for a CRRA investor is e.g. solved in Merton (1971). The optimal strategy is a CP strategy with portfolio weight

$$\pi_t = \frac{a_t}{\gamma \sigma^2},$$

where the expected excess return a_t is constant. The results of the Monte-Carlo simulation show that the utility differences between (optimal) BH and CP strategies are very small.[11] This finding is in line with previous papers, see e.g. Rogers (2001) or Branger, Breuer, and Schlag (2009).

Furthermore, the optimal PI strategies are also only slightly worse than the overall optimal strategy. Note, however, that the investor chooses a rather low protection level. In the case of the CPPI, e.g., he sets the floor equal to the minimal value of 50, and invests 140% of the initial cushion and thus around 70% of his initial wealth into the stock. He therefore tries to mimick the optimal CP strategy as closely as possible, and utility losses would be larger if we forced him to chose a protection level close to his initial wealth.

In the SV model, the overall optimal strategy is not in the set of strategies we consider. Among the strategies described in Section 2.3, CP is optimal for $\gamma < 1.7$ and for $\gamma > 5$, while PP133 is optimal for a relative risk aversion between 1.7 and 5. However, the difference in certainty equivalents is at most 16 basis points, which is very small compared to the overall gain from trading stocks and bonds of around 10%. And again, the rather good performance of PI strategies can be attributed to the fact that the investor chooses protection levels well below his initial wealth.

The results for the SVJ model are qualitatively similar. Again, CP is not the overall optimal strategy, but performs rather well in the restricted set of strategies. In the simulation, none of the other strategies is superior to CP by more than fractions of a basis point.

[11] In the following, we will always refer to the optimal strategy within a class of strategies, not to all strategies from this class.

3.2 CPT investor

In the following section we look at the optimal strategies for a CPT investor and analyze the sensitivity of the results with respect to changes in loss aversion, curvature and probability weighting. In particular, we want to identify the features of the CPT utility function that are vital to explain the attractiveness of PI. Our benchmark is given by the median decision maker of Tversky and Kahneman (1992) with a reference point equal to the initial wealth of 100.

3.2.1 Black-Scholes

We first consider the model of Black-Scholes. The certainty equivalents for the optimal CP and the overall optimal strategy in the benchmark case are given in the first line of Table 3. The overall optimal choice is a PP133 strategy which leads to a certainty equivalent of around 110. The protection level slightly exceeds the reference point, so that the strategy is now – in contrast to the CRRA case – truly a PI strategy. The density of the terminal wealth for this strategy is given in the upper graph in Figure 1. It shows that there is a substantial probability of exercising the put and thus ending up with a terminal wealth equal to the protection level. On the other hand, there is also significant upside potential.

The optimal CP strategy is characterized by a small weight for the stock of just 9%. This conservative choice, with a loss probability below 0.1%, ensures that there are hardly any losses. However, it also limits the upside potential severely, as can again be seen in the upper graph in Figure 1. The probability for a terminal wealth above 110 is basically equal to zero. Consequently, the certainty equivalent is around 550 bp lower than for the optimal PP133 strategy. This is in stark contrast to the case of the CRRA investor for whom the differences in performance were negligible.

The optimal CPPI strategy is given by the (maximum) multiplier of 10 and a terminal guarantee level of 100. The certainty equivalent of this strategy is by only 15 bp smaller than the one of the optimal PP133 strategy. Figure 1 shows that the density of the terminal distribution is similar to that of the PP133, and that both PI strategies have a much higher upside potential than the less attractive CP strategy. The lower graph in Figure 1 compares the contributions of the different wealth levels to the final certainty equivalent for various specifications of the preferences in the gain domain.[12] With $\alpha = 1$ and $\delta^+ = 1$, the certainty equivalent of the PI strategies is basically equal to the expected terminal payoff, because the probability of ending

[12]The accumulation graph compares the performance of optimal CPPI and PP133 strategies for

up below the reference point of 100 is very small. For such linear preferences in the gain domain the certainty equivalent is slightly larger for PP133 than for CPPI. Concavity in the gain domain ($\alpha = 0.88$) decreases both certainty equivalents, and has a larger impact on the protective put. Finally, probability weighting ($\delta^{+} = 0.61$) benefits the protective put more than the CPPI strategy.

PI is thus attractive for a CPT investor. To see which elements of the preferences are the main drivers of this result, we perform a sensitivity analysis with respect to CPT's core components. The results are given in Table 3.

The sensitivity analysis *with respect to loss aversion* shows that increasing the loss aversion k beyond the benchmark value does not have an impact on the optimal strategy and on the associated utility gain. For $k = 2.25$, the investor already chooses strategies with no losses (protective put) or hardly any losses (if restricted to CP). Loss aversion levels below 2.25 ($k < 2.25$) lead to a way higher risk taking in case of CP (133% stock), since losses are penalized less.[13] Among the PI strategies the investor switches from PP133 (for high loss aversion) to CPPI (for low loss aversion), which can be attributed to the more pronounced right skewness of the terminal wealth for CPPI (see Figure 1). With a smaller loss aversion, the investor chooses a protection level below the reference point. The corresponding increase in the flexibility via a larger cushion allows him to achieve heavily right-skewed distributions and a larger CE. Since a smaller loss aversion also implies that CP strategies become more attractive, the difference in CEs between PI and CP decreases. When downside protection is less important the investor is able to take advantage of stock market within the confines of the CP strategy.[14]

Next we analyze the sensitivity *with respect to changes in the curvature of the value function.* Since the large loss aversion induces the investor to avoid all strategies with a significant probability of losses anyway, changing the curvature of the value function in the loss domain hardly has some impact on utility gains. Linearity (no

various parameterizations of CPT preferences in the gain domain. The certainty equivalent for a specific value of wealth (x-axis), e.g. 150, quantifies the value of the certainty equivalent under the constraint that all contributions in utility from outcomes above 150 are set to zero. Note that, due to the rank-dependence of probability weighting, this approach is methodologically different from resetting outcomes above 150 to exact 150 or to the initial wealth of 100.

[13] Note that the utility function is convex in loss region.

[14] A similar effect is observed in Dierkes, Erner, and Zeisberger (2009) for longer investment horizons of up to 7 years. Since the probability of a wealth level below the reference point is the lower the longer the investment horizon, standard levels of loss aversion (without probability weighting) can no longer explain the demand for PI. On the other hand, an extreme overweighting of high gains (without loss aversion) that occur in particular for CPPI still makes PI superior.

curvature) in the gain domain increases rewards for gains and leads to maximum risk taking for the CP strategy. For PI strategies, the investor again switches from PP133 to CPPI with a lower protection level. The probability for very high gains is larger for the latter strategy, and gambling for high gains pays off more with linearity of the positive part of the value function. In the case of a linear value function with a kink at the reference point, the investor still holds 133% stocks in a CP strategy. The reason is that the high reward for large gains remains overweighted due to probability weighting. At the same time, there is no more convexity in the loss domain, so that the investor strongly prefers right-skewed PI to symmetric CP, and the gap between the two strategies reaches its maximum value of around 590 basis points.

The sensitivity analysis *with respect to probability weighting* reveals some interesting properties of the CPT model. For the four cases of probability weighting (for gains and losses, for gains only, for losses only, and no probability weighting) Figure 2 shows the density functions of the terminal wealth from the optimal CP strategies and the weighted density functions. When there is no probability weighting for losses, (extreme) losses are penalized less. This has an effect similar to a lower loss aversion: the optimal weight in a CP strategy increases to 133%. Among the PI strategies, PP133 remains optimal with a protection level that slightly decreases to 100%. Without probability weighting for gains, the optimal weight in a CP strategy increases from 9% to 16%, which is counter-intuitive at a first glance. Here, a trade-off between two opposing effects is the decisive factor. Firstly, large gains are no longer overweighted, which would lead to a lower investment into the stock. Secondly, probability weighting reduces the total probability mass attributed to gains and thus replaces positive utility by zero utility. Consequently, eliminating probability weighting makes gains more attractive. Here, the second effect dominates, and the investor chooses a larger investment into stocks. When there is no probability weighting at all, the optimal weights in a CP strategy increase to 48%. Due to the lack of negative returns, the performance of the optimal PI strategy (PP133) is the same as in the case of no probability weighting for gains.

With loss aversion and probability weighting, the CPT investor strongly prefers PI strategies to CP strategies with a gain in certainty equivalent of around 550 basis points. If there is only loss aversion (but no probability weighting) or if there is only probability weighting (but no loss aversion), then PI strategies are still superior to CP strategies, but the utility gain decreases sharply to around 80 basis points. If there is neither loss aversion nor probability weighting, then the gain from PI strategies as compared to CP strategies drops to basically zero.

Finally, we check the sensitivity of our results *with respect to the functional form of probability weighting*. The changes are only marginal. We observe a slightly higher stock weight for CP and also slightly lower protection levels for PP133. The intuition runs akin to completely deactivating probability weighting, but effects are much smaller in magnitude. All other sensitivity analyses produce very similar results as above. Therefore the presented results can be regarded as robust to changes in the exact specification of the inversely S-shaped probability weighting function.

3.2.2 Stochastic Volatility

Next, we consider the stochastic volatility model of Heston (1993). The investment opportunity set is now stochastic, and in a complete market, the investor profits from the additional risk premium on volatility risk. In an incomplete market where the investor can only trade in stocks, bonds, and protective puts and is furthermore restricted to strategies with constant parameters the results are, however, rather similar to the case of the Black-Scholes model. Again, a CPT investor profits from PI with a gain in the certainty equivalent of around 460 basis points. Table 4 gives the certainty equivalents in the benchmark case and for the sensitivity analysis.

Figure 3 compares the densities of the stock price in the BS model and in the SV model. Stochastic volatility leads to fatter tails, and due to the negative correlation between stock returns and volatility innovations, the distribution becomes more left-skewed. While the investor profits from the larger upside potential, the increased downside potential and the larger left-skewness lower his utility. With CPT preferences, the investor chooses strategies which nearly avoid losses, so that the first effect dominates. Consequently, the certainty equivalents are larger than in the BS case.

Furthermore, the stock becomes more attractive to the CPT investor and the CP investment increases to 133% in all cases but the one where $\alpha = 0.88$ and $\beta = 1.0$. In this case, the punishment for losses without an offsetting effect for gains precludes the investor from leveraging. Among the PI strategies, PP133 is nearly always optimal, with the exception of the cases with no loss aversion or no curvature for gains. The reason is that PP133 is better suited than CPPI to cope with the larger downside potential of stocks that arises from a heavier left tail and a larger left-skewness. As can be seen from the lower graph in Figure 4, PP133 derives its advantage from one additional percentage point of expected return and a return distribution that incurs lower losses (in certainty equivalent terms) due to curvature.

3.2.3 Stochastic Volatility and Jumps

In the SVJ model, there are also jumps in the stock price. This increases the risk of reaching very low stock price levels, and it leads to gap risk in CPPI strategies as soon as the multiplier exceeds one. Nevertheless, the overall results and explanations are again very similar to the BS-case. Table 5 gives the results of the benchmark case and of the sensitivity analyses for the SVJ model.

The main difference to the BS case occurs for the case of a linear value function ($\alpha = 1.0$ and $\beta = 1.0$). The linear penalty for losses raises the investor's concern for downward jumps, in particular since loss aversion and probability weighting are still effective, so that he decides to invest only 16% into the stock (as opposed to 133% in the BS-model).

For PI strategies, CPPI generally performs slightly better than PP133. While a CPPI strategy leads to a higher potential for very high gains, it also suffers from the gap risk due to jumps. In an OBPI strategy, on the other hand, the probability of falling below the protection level is zero. However, the protective put includes a large risk premium for downward jumps, which may exceed the investor's willingness-to-pay.[15] A closer look at the comparative performance in the lower graph in Figure 5 reveals that, contrary to the SV model, the CPPI strategy offers nearly one additional percentage point of expected return and leads to a return distribution with losses due to curvature that are on par with the PP133 strategy. The better accommodation of probability weighting is insufficient for PP133 to close the gap on CPPI. Finally, note that the gap risk is less severe for a CPT investor than for a CRRA investor due to the convexity of the value function in the loss domain.

4 Conclusion

Empirical evidence shows that portfolio insurance strategies are attractive for investors. This finding is puzzling for a CRRA investor, who prefers constant proportion strategies. Even if portfolio insurance strategies lead to only small losses in utility as compared to the optimal strategies, the mediocre performance of these strategies cannot explain their popularity. Furthermore, the investor should choose

[15] See also Driessen and Maenhout (2007) who show that the investor often opts for a short position in puts to earn this large risk premium.

protection levels well below his initial wealth, which is largely at odds with the intuition behind portfolio insurance. This does not only hold true in the model setup of Black-Scholes, but also in models with stochastic volatility and jumps in stock prices.

In contrast to a CRRA investor, a CPT investor profits significantly from following a portfolio insurance strategy. The increase in the certainty equivalent due to trading more than doubles if the investor switches from a constant proportion strategy to a portfolio insurance strategy. In line with intuition, he will choose a protection level slightly above his reference point. A sensitivity analysis shows that both loss aversion and probability weighting are vital to explain the strong preference for portfolio insurance. If only one of these two features of CPT preferences is present, portfolio insurance is still attractive, but the utility gains due to portfolio insurance drop significantly. The overall attractiveness of portfolio insurance holds in models with and without stochastic volatility and jumps. Yet, the choice between constant proportion portfolio insurance and option based portfolio insurance depends on the return generating process.

Portfolio insurance strategies give rise to right-skewed return distributions. The empirically observed attractiveness of these strategies can thus be interpreted as evidence for the existence of a group of investors in the market who are both loss averse and exhibit probability weighting.[16]

[16] In addition to field evidence of a preference for right-skewness (Kapadia (2006), Mitton and Vorkink (2007)), it was shown by Brünner, Levínský, and Qiu (2007) and Vrecko, Klos, and Langer (2009) in experimental studies that subjects act skewness seeking in choice tasks.

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Param.	BS	SV	SVJ
b	0.1119	0.1119	0.1250
$\kappa^{\mathbb{P}}$		5.7960	3.2256
$\theta^{\mathbb{P}}$	0.0228	0.0227	0.0205
σ_V		0.3528	0.2404
ρ		-0.4000	-0.4668
$\mu_S^{\mathbb{P}}$			-0.0259
$\sigma_S^{\mathbb{P}}$			0.0407
$\lambda^{\mathbb{P}}$			1.5120
η_V		1.26	1.512
$\lambda^{\mathbb{Q}}$			$\lambda^{\mathbb{P}}$
$\mu_S^{\mathbb{Q}}$			-0.0491
$\sigma_S^{\mathbb{Q}}$			0.0994

Table 1: **Parameters under the $\mathbb{P}$-measure and market prices of risk**

The table gives the parameters under the objective measure as estimated by Eraker, Johannes, and Polson (2003) (EJP) for the Black-Scholes model (BS), the Heston-model (SV), and the model with stochastic volatility and jumps in the stock price (SVJ). It also gives the market prices of risk as estimated by Broadie, Chernov, and Johannes (2007) based on these $\mathbb{P}$-parameters. All parameters are given as annual decimals.

	Constant Proportion	**Buy & Hold**	**CPPI**	**Protective Put**	**Protective Put 133**
BS	0.75 106.71	0.75 106.71	50.0/1.4 106.70	81.0/80.97 106.48	102.0/94.49 106.08
SV	1.18 110.25	1.17 110.26	50.0/10.0 110.15	73.0/72.74 110.13	89.0/81.75 110.41
SVJ	0.65 106.63	0.65 106.63	52.0/1.3 106.63	63.0/63.05 106.00	103.0/93.28 105.07

Table 2: **CRRA investor**

The table gives - in the first line for each model - the portfolio weights for the optimal constant proportion and the optimal buy-and-hold strategies, the optimal floor and multiplier for the CPPI strategy, and the optimal strike and protection level for the protective put strategies. It also gives - in the second line - the certainty equivalents. The parameters are given in Table 1, and relative risk aversion is set equal to $\gamma = 3$.

α	β	k	PW type	δ^+	δ^-	Best CP	CE	Best overall	CE	Gap (bp)
0.88	0.88	2.25	T/K	0.61	0.69	CP_0.09	104.66	PP_114_100.457	110.21	554.40
		1				CP_1.33	110.84	CPPI.90_10.0	111.60	76.86
		1.625				CP_1.33	107.13	CPPI.98_10.0	110.24	311.41
		2.875				CP_0.09	104.66	PP_114_100.457	110.21	554.64
0.88	1					CP_0.09	104.66	PP_114_100.457	110.21	554.43
1	0.88					CP_1.33	110.39	CPPI.94_10.0	112.61	222.30
1	1					CP_1.33	106.40	CPPI.98_10.0	112.30	589.86
				0.61	1	CP_1.33	107.46	PP_114_100.457	110.21	274.94
				1	0.69	CP_0.16	105.24	PP_113_100.089	106.62	138.08
				1	1	CP_0.48	105.75	PP_113_100.089	106.62	86.79
1	1	1				CP_1.33	113.64	CPPI.90_10.0	114.68	104.08
1	1			1	1	CP_1.33	107.46	PP_113_100.013	107.75	28.71
		1		1	1	CP_1.33	110.50	PP_58_42.089	110.51	0.88
1	1	1		1	1	CP_1.33	112.32	CPPI.50_10.0	112.32	0.08
			R/W			CP_0.13	104.76	PP_113_100.313	109.55	479.10

Table 3: **Sensitivity analyses - Black-Scholes model**

The table presents the numerical results of the sensitivity analyses with respect to loss aversion, curvature and probability weighting in the Black-Scholes model. k is the loss aversion parameter, α and β describe the curvature of the value function in the gain and in the loss domain, and δ^+ and δ^- give the probability weighting for gains and losses. 'Best CP' gives the portfolio weight of the best constant proportion strategy with the certainty equivalent CE, 'Best overall' gives the best overall strategy with its CE, where 'PP_x_y' denotes the PP133 strategy with a strike price of x and a resulting protection level of y and 'CPPIx_y' denotes the CPPI strategy with floor $x \times 100$ and multiplier y. Gap is the difference in the certainty equivalents between the best CP strategy and the best overall strategy.

α	β	k	PW type	δ^+	δ^-	Best CP	CE	Best overall	CE	Gap (bp)
0.88	0.88	2.25	T/K	0.61	0.69	CP_1.33	108.19	PP_111_100.200	112.82	462.62
		1				CP_1.33	114.50	CPPI.76_9.7	114.54	4.41
		1.625				CP_1.33	111.29	PP_111_100.200	112.82	152.82
		2.875				CP_1.33	105.23	PP_111_100.200	112.82	758.71
0.88	1					CP_0.24	105.14	PP_111_100.200	112.82	768.07
1	0.88					CP_1.33	114.69	CPPI.90_10.0	115.22	53.32
1	1					CP_1.33	111.34	PP_111_99.982	114.79	345.06
				0.61	1	CP_1.33	112.74	PP_111_100.200	112.82	7.84
				1	0.69	CP_1.33	107.19	PP_111_100.026	109.97	277.95
				1	1	CP_1.33	111.68	133	111.71	2.86
1	1	1				CP_1.33	117.35	CPPI.84_10.0	117.58	22.92
1	1			1	1	CP_1.33	113.81	CP_1.33	113.81	0.00
		1		1	1	CP_1.33	115.52	CP_1.33	115.52	0.00
1	1	1		1	1	CP_1.33	117.25	CP_1.33	117.25	0.00
			R/W			CP_1.33	107.73	PP_112_100.574	111.88	414.96

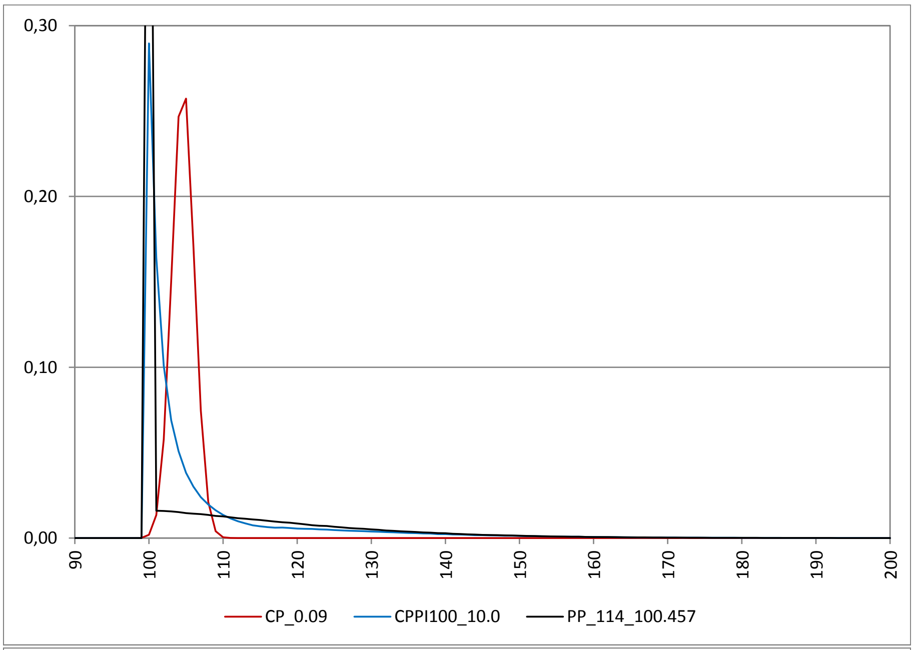
Table 4: **Sensitivity analyses - SV model**

The table presents the numerical results of the sensitivity analyses with respect to loss aversion, curvature and probability weighting in the Stochastic Volatility model. k is the loss aversion parameter, α and β describe the curvature of the value function in the gain and in the loss domain, and δ^+ and δ^- give the probability weighting for gains and losses. 'Best CP' gives the portfolio weight of the best constant proportion strategy with the certainty equivalent CE, 'Best overall' gives the best overall strategy with its CE, where 'PP_x_y' denotes the PP133 strategy with a strike price of x and a resulting protection level of y and 'CPPIx_y' denotes the CPPI strategy with floor $x \times 100$ and multiplier y. Gap is the difference in the certainty equivalents between the best CP strategy and the best overall strategy.

α	β	k	PW type	δ^+	δ^-	Best CP	CE	Best overall	CE	Gap (bp)
0.88	0.88	2.25	T/K	0.61	0.69	CP_0.00	104.60	CPPI100_10.0	109.54	493.99
		1				CP_1.33	110.48	CPPI.88_10.0	111.27	79.56
		1.625				CP_1.33	106.19	CPPI.98_10.0	109.68	349.29
		2.875				CP_0.00	104.60	CPPI100_10.0	109.45	485.10
0.88	1					CP_0.00	104.60	CPPI100_10.0	109.52	491.33
1	0.88					CP_1.33	109.57	CPPI.96_10.0	112.00	242.46
1	1					CP_0.16	104.64	CPPI.98_10.0	111.69	704.76
				0.61	1	CP_1.33	106.92	CPPI100_10.0	109.78	286.16
				1	0.69	CP_0.12	105.11	CPPI100_8.4	106.24	112.56
				1	1	CP_0.45	105.68	CPPI100_10.0	106.40	72.03
1	1	1				CP_1.33	113.18	CPPI.88_10.0	114.42	124.14
1	1			1	1	CP_1.33	107.24	CPPI.98_9.7	107.36	12.69
		1		1	1	CP_1.33	111.04	133	111.09	4.99
1	1	1		1	1	CP_1.33	112.85	CP_1.33	112.85	0.00
			R/W			CP_0.08	104.66	CPPI100_10.0	108.67	400.66

Table 5: **Sensitivity analyses - SVJ model**

The table presents the numerical results of the sensitivity analyses with respect to loss aversion, curvature and probability weighting in the Stochastic Volatility and Jumps model. k is the loss aversion parameter, α and β describe the curvature of the value function in the gain and in the loss domain, and δ^+ and δ^- give the probability weighting for gains and losses. 'Best CP' gives the portfolio weight of the best constant proportion strategy with the certainty equivalent CE, 'Best overall' gives the best overall strategy with its CE, where 'PP_x_y' denotes the PP133 strategy with a strike price of x and a resulting protection level of y and 'CPPIx_y' denotes the CPPI strategy with floor $x \times 100$ and multiplier y. Gap is the difference in the certainty equivalents between the best CP strategy and the best overall strategy.



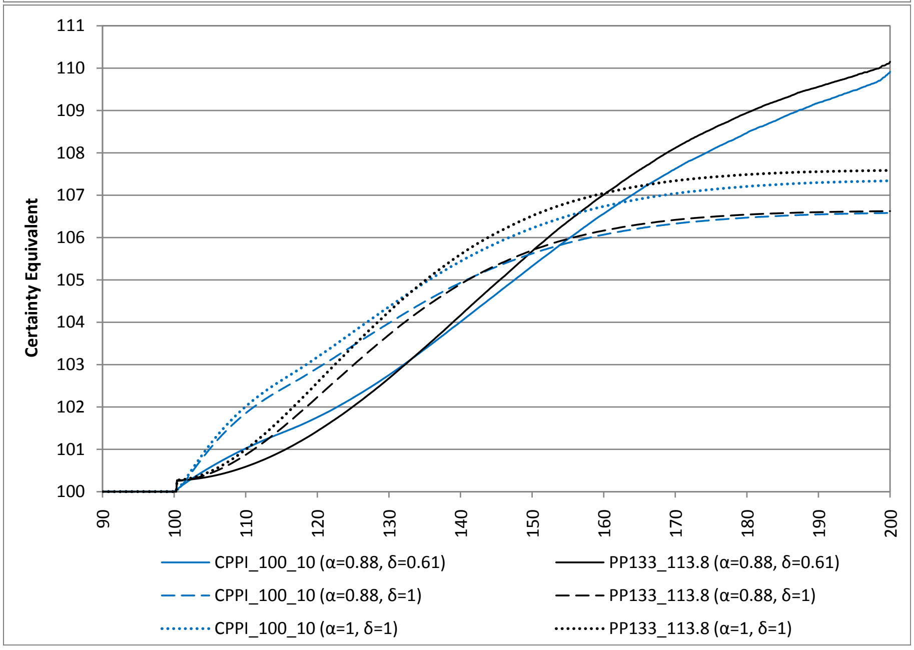


Figure 1: **Terminal wealth distributions for best CP, CPPI and PP133 strategies - Black-Scholes model**

The upper graph shows the histogram of the terminal wealth distributions for the best CP (CP_0.09), CPPI (CPPI100_10.0) and PP133 (PP_114_100.457) strategies in the Black-Scholes model. The lower graph compares the accumulation for the optimal portfolio insurance strategies for various parameterizations of CPT preferences in the gain domain. The parameterization in the loss domain remains standard (β=0.88, δ^-=0.69).The dotted line shows how expected values are accumulated (α=1, δ^+=1), the dashed line visualizes the effects of curvature (α=0.88, δ^+=1), and the solid line quantifies the contribution of different outcomes to the final certainty equivalent of a strategy (α=0.88, δ^+=0.61).

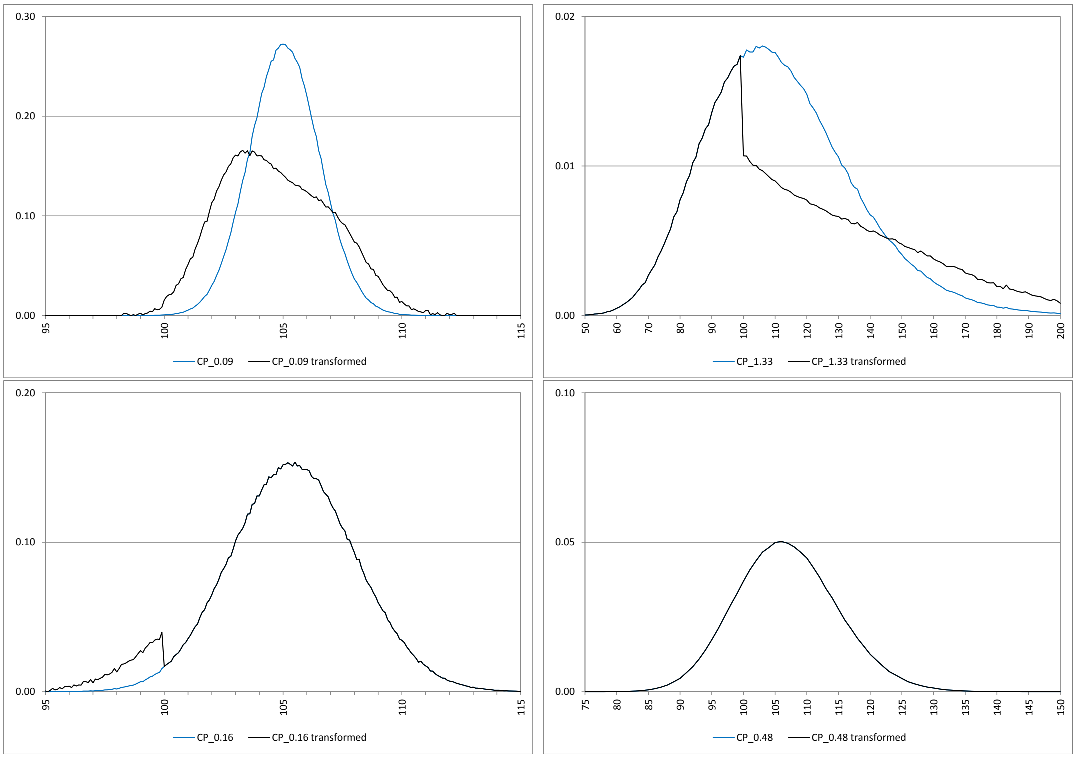


Figure 2: **Effects of probability weighting on terminal wealth distributions - Black-Scholes model**

The figure shows the effects of probability weighting on optimal CP strategies in case of full probability weighting (top left graph), probability weighting for gains only (top right graph), probability weighting for losses only (bottom left graph) and no probability weighting (bottom right graph).

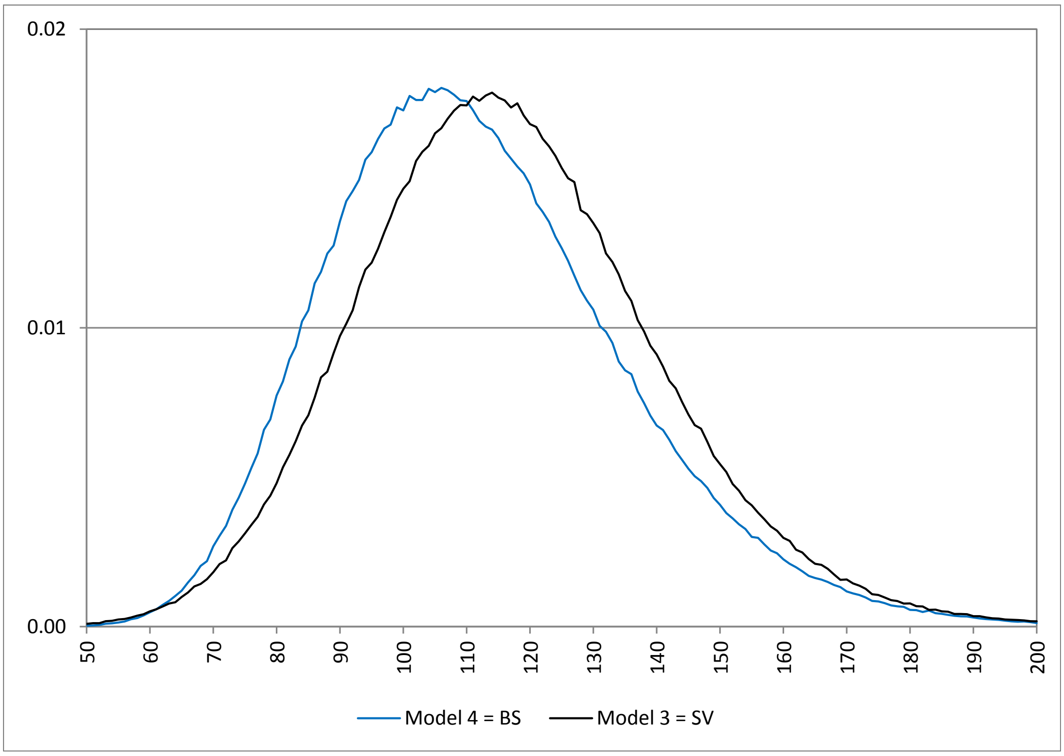


Figure 3: **Terminal wealth distributions for BS and SV**

The figure shows the density of terminal wealth for a CP_1.00 strategy (i.e. for an investment into stocks only) in the BS model and in the SV model.

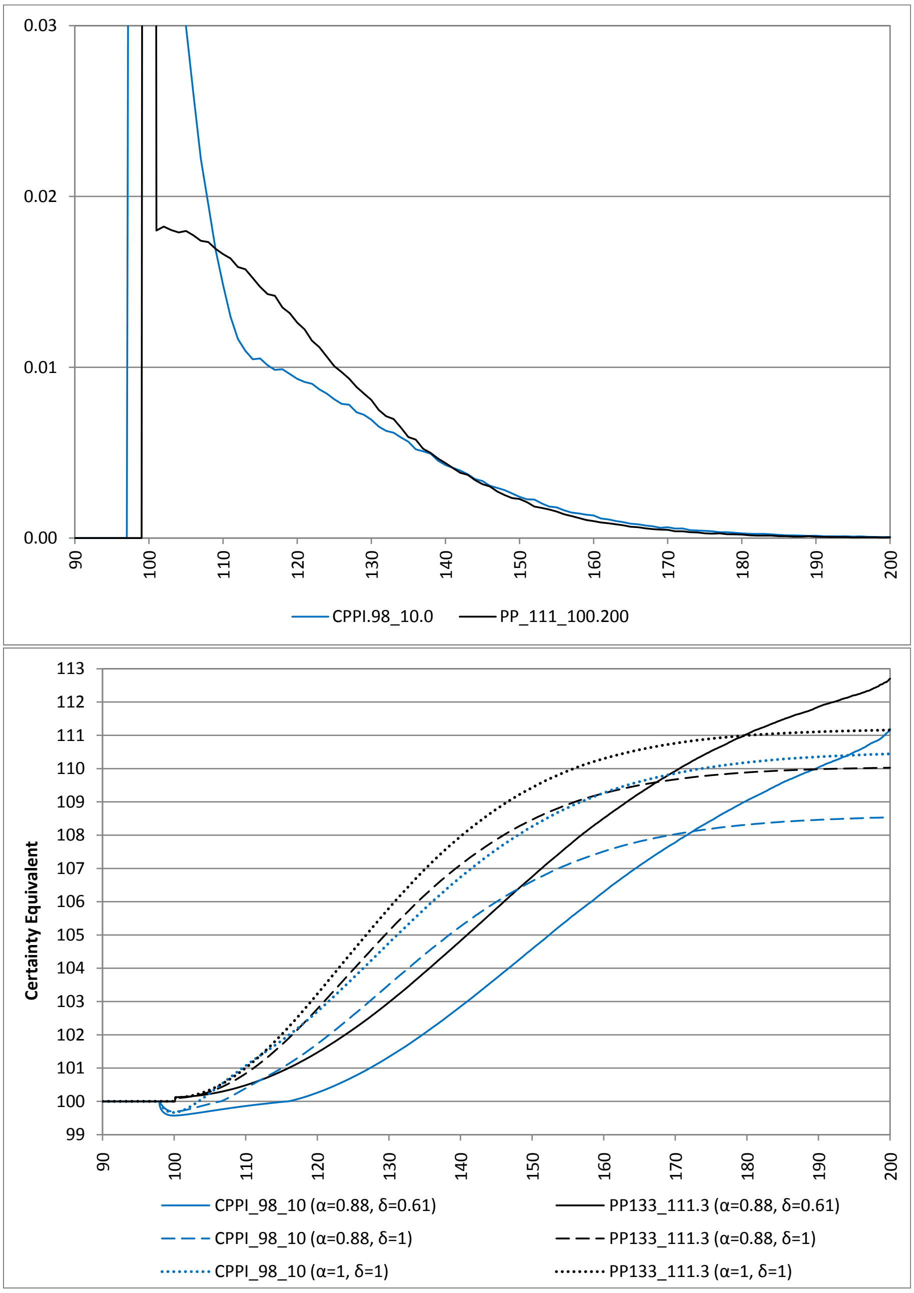


Figure 4: **Terminal wealth distributions for best CPPI and PP133 strategies - Stochastic Volatility model**

The upper graph shows the histogram of the terminal wealth distributions for the best CPPI (CPPI.98_10.0) and PP133 (PP_111_100.200) strategies in the SV model. The lower graph compares the accumulation for the optimal portfolio insurance strategies for various parameterizations of CPT preferences in the gain domain. The parameterization in the loss domain remains standard (β=0.88, δ^{-}=0.69). The dotted line shows how expected values are accumulated (α=1, δ^{+}=1), the dashed line visualizes the effects of curvature (α=0.88, δ^{+}=1), and the solid line quantifies the contribution of different outcomes to the final certainty equivalent of a strategy (α=0.88, δ^{+}=0.61).

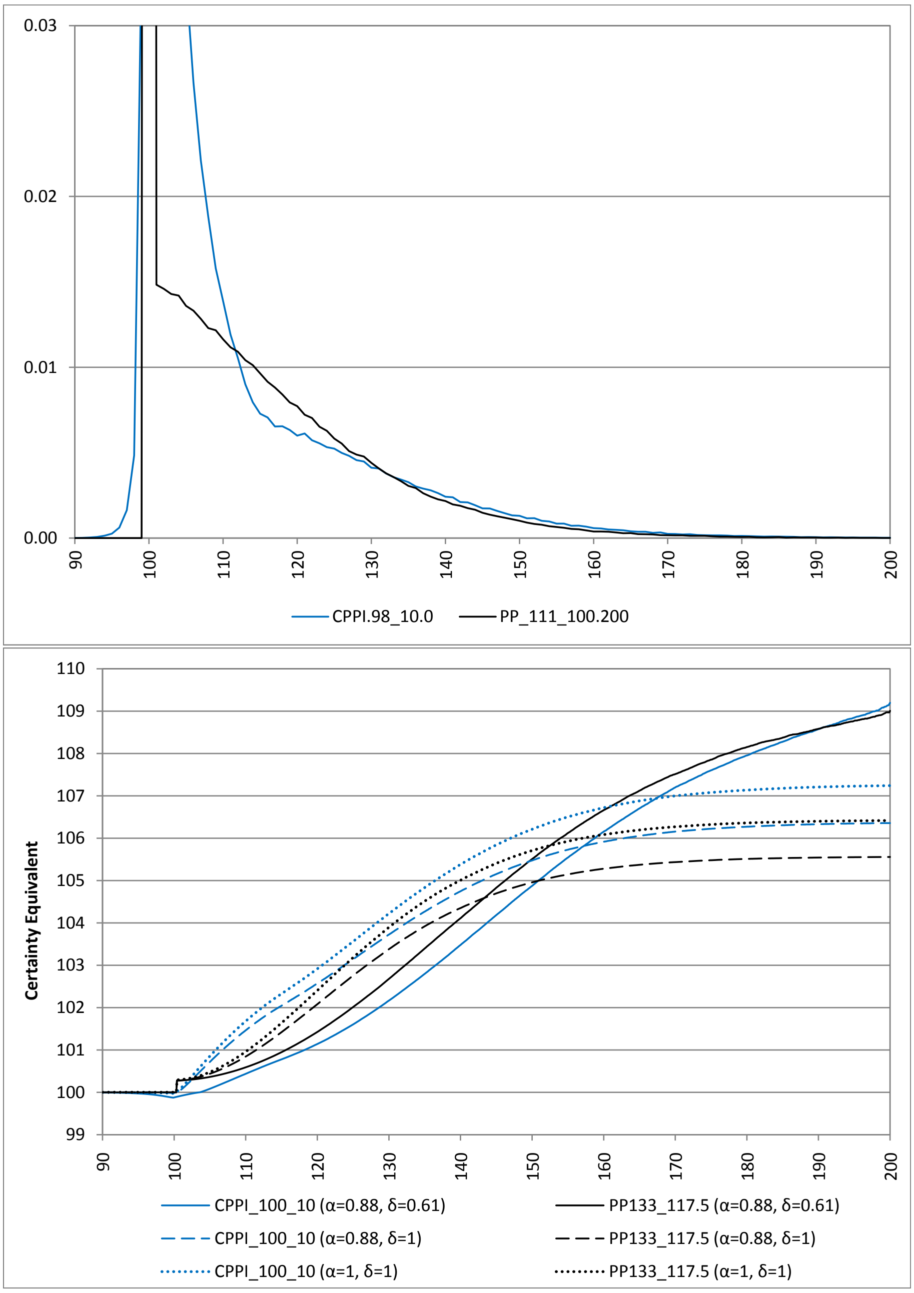


Figure 5: **Terminal wealth distributions for best CPPI and PP133 strategies - SVJ model**

The upper graph shows the histogram of the terminal wealth distributions for the best CPPI (CPPI.98_10.0) and PP133 (PP_111_100.200) strategies in the SVJ model. The lower graph compares the accumulation for the optimal portfolio insurance strategies for various parameterizations of CPT preferences in the gain domain. The parameterization in the loss domain remains standard (β=0.88, δ^{-}=0.69). The dotted line shows how expected values are accumulated (α=1, δ^{+}=1), the dashed line visualizes the effects of curvature (α=0.88, δ^{+}=1), and the solid line quantifies the contribution of different outcomes to the final certainty equivalent of a strategy (α=0.88, δ^{+}=0.61).